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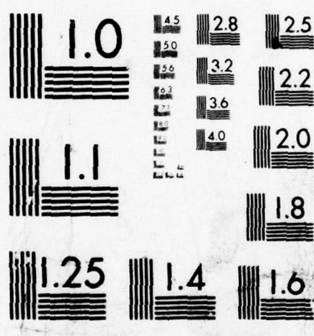

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TEAMS WITH VARIABLE PRECISION INFORMATION STRUCTURES: A MODEL FOR ORGANIZATIONAL FORM*

by

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Introduction

The theory of teams was originally formulated to model organizations [1]. The usual translation is to interpret the information structure of a team as the structure of the corresponding organization. Although this translation task is admittedly difficult, especially for the static information structures considered in this paper, certain insights can be gained by examining the optimal information structure for a given problem. In particular, the derived information structures can be interpreted as guidelines for coordination of effort versus specialization of effort. However, the model considered here gives no insight into how this coordination of effort may be accomplished. The question of whether the coordination is best accomplished through a hierarchy, a matrix organization, a "Steering Committee," or some other device must be answered elsewhere. This model will suggest the extent of coordination desirable for optimal performance.

The usual formulation of the team information problem--who knows what?--regards information as a collection of discrete objects [1]. Attempts to solve for optimal information structures under such an assumption are quickly stymied by problems of combinatorial complexity; additionally, analytic results in this case are difficult to obtain.

Another motivation for scrapping the discrete information approach is the translation of information structures to guidelines for organizational coordination. In contrast to the impression obtained from examining organization charts, coordination is not a "yes-or-no" question, but rather a question of degree. Various means are at the organization designer's disposal to effect the desired degree of coordination. The hierarchical structure evident in most organization charts is but one way to implement coordination. Committees,

internal newsletters, distribution of memoranda and the "grapevine" also serve to provide coordination.

The principal features of the model developed here is that the information sets for the decision makers contain base information viewed through noisy information channels. The amount of noise on the channel for each decision maker is a parameter to be optimized. The total amount of noise that a decision maker must accept over all his pieces of information is a constraint of the model. The practical motivation treats the information acquisition process as reading samples or "memos" about the piece of information desired--the more memos read, the more accurate the information or the less noisy the information channel through which the information is read. Practically, there is a limit to the number of memos that an agent can read, hence the constraint on the total noise that the decision maker must accept. Additionally, it is clear in real life that it may be more difficult to read some other department's memos than ones own memos, hence the model contains parameters to capture this difficulty.

Each decision maker is then characterized by two sets of parameters: one which specifies the least amount of noise that he will be forced to accept, or the most amount of precision he can spend; and the other set specifies the tradeoff between reading his memos and some other department's memos.

Alternatively, these constraints may be interpreted as specifying the cost of acquiring or, more properly, processing information. One can either assign a shadow price to precision, as is done here by use of a constraint on total precision expendable, or can specify an arbitrary price for precision.

The model is completed by a specification of the payoff function--a mapping from the set of possible states of nature and actions of team members to a scalar. This scalar is the team payoff; the function is supposed to

embody the environment that the organization faces. In a crude sense, one may determine the best organization for a given environment by solving the corresponding model problem--finding the optimal noise levels on pieces of information as a function of the decision makers' capabilities and the team payoff function.

In this note the usual engineering expedient of using a quadratic payoff function with Gaussian states of nature and information linear in the state of nature is taken. Additionally, the information sets are constrained to be static, i.e., no decision maker depends on another's action for his information. In the sequel the general Linear-Quadratic-Gaussian model is developed and two simple cases are formulated and solved.

1. Model

$$\min_{p_{ij}} [\min_{u_k} \underline{u}'Q\underline{u} + 2\underline{u}'S\underline{x}] \tag{1.1}$$

$i=1,\ldots,n;\ j=1,\ldots,n;\ k=1,\ldots,n$

$$x \sim N(\underline{0}, I_n) \tag{1.2}$$

$$u_i = f_i(H\underline{x} + (w_{i1},\ldots,w_{in})') \tag{1.3}$$

$$w_{ij} \sim N(0, p_{ij}^{-1}) \tag{1.4}$$

$$\Sigma b_{ij} p_{ij} \leq c_i \tag{1.5}$$

$$b_{ii} \equiv 1$$

All random variables are mutually independent.

A nested optimization problem is specified in (1.1). The inner optimization problem represents the firm's optimal response to the environment, specified by the payoff function, under a given information structure. The outer optimization problem represents the organization designer's problem: determine the optimal information structure given the payoff and the constraints on the decision makers.

The payoff function is a standard quadratic payoff function specified by the matrices Q and S. While this selection may not be the most desirable for economic work, it does allow easy solution of the problem at hand.

The information structure is specified in (1.3), (1.4) and (1.5). Each decision maker receives the same basic set of n pieces of information corrupted by assignable amounts of noise. Thus decision maker i receives information j through an additive Gaussian white noise channel: the corrupting noise w_{ij} has a precision p_{ij} or, equivalently, variance p_{ij}^{-1}. Note that this formulation permits the possibility that decision maker i does not use the jth piece of information: p_{ij} is then set to zero.

The characteristics of the decision maker are specified in (1.5). A decision maker is parameterized by c_i and the set $b_{i1},\ldots,b_{in}$. The c_i represent the constraints on the total amount of precision that the decision makers can allocate. Intuitively, decision maker i can read no more than a fixed number of memos. The b_{ij} represent the difficulty that decision maker i has reading a memo concerning information j. For simplicity, decision maker i has unit difficulty reading information i.

Alternatively, this model may be interpreted as specifying a cost of acquiring or processing information about the state of nature. The costs may be taken to be the shadow prices due to the constraint on total precision available, or may be explicitly given. In the latter case, (1.1) would take

the form

$$\min_{p_{ij}} [\min_{u_k} \underline{u}'Q\underline{u} + 2\underline{u}'S\underline{x}] + \Sigma b_{ij} p_{ij} \tag{1.6}$$

One would delete (1.5) and interpret the b_{ij} as the price of precision--how much must decision maker i pay for one unit of precision for information j.

The inner optimization problem in (1.1) or (1.6) may be solved [2]. This solution yields the following more concise statement of the organization designer's problem

$$\min_{p_{ij}} (S_1;\ldots;S_n)^T H^T \left[Q \otimes HH^T + \mathrm{Diag}\left(\frac{q_{11}}{p_{11}}, \ldots, \frac{q_{11}}{p_{1n}}, \ldots, \frac{q_{nn}}{p_{n1}}, \ldots, \frac{q_{nn}}{p_{nn}} \right) \right]^{-1} .$$

$$H(S_1;\ldots;S_n) \tag{1.7}$$

where S_i is the ith row of S and $Q \otimes HH^T$ is the Kronecker Product of Q and HH^T, i.e., it is an n by n block matrix, the i_jth block (n by n) given by $q_{ij}HH^T$.

This statement of the problem does not readily admit an explicit solution; the rest of the paper is dedicated to two special cases and their solution.

2. One-person Team

A one-person formulation of the organization problem is interesting because it isolates a threshold effect inherent in the solution of the model. Consider the following nested optimization problem

$$\min_p [\min_u (x + u)^2] + bp \tag{2.1}$$

This problem is simply the problem stated in (1.6) with Q = 1 and S = 1. Note that this form of the problem--price of precision explicitly given by b--

is the only reasonable one: a constraint on total precision allocated to one piece of information is meaningless.

Conceptually, the inner optimization problem may be solved for given values of p; the outer optimization problem may then be solved to find the optimal precision for the corrupting noise by minimizing the sum of the cost of precision and the function derived by solving the inner optimization problem. This algorithm is precisely the one implied by the form of the problem in (1.7) and applies to more general statements of the problem.

The solution of the inner optimization problem, obtained by standard techniques, is

$$J(p) = (1 + p)^{-1} \quad . \tag{2.2}$$

The outer optimization problem becomes

$$\min_{p}[J(p) + bp] \; . \tag{2.3}$$

The solution to the nested optimization problem in (2.1) is then

$$p = \begin{cases} b^{-1/2} - 1 & b \leq 1 \\ 0 & b \geq 1 \end{cases} \tag{2.4}$$

The interesting point to note is the threshold effect--no information, zero precision, is demanded if the price is too high. This effect is hardly surprising; it is similar to phenomena present in calculations of stopping rules in sequential sampling problems. This effect is noted here because it appears in more general settings of the problem.

3. Two-person Symmetric Team

Consider the following problem

$$Q = \begin{bmatrix} 1 & q \\ q & 1 \end{bmatrix}, \; S = I_2, \; H = I_2 \tag{3.1}$$

$$c_1 = c_2 = c, \; b_{12} = b_{21} = b, \; b_{11} = b_{22} = 1 \tag{3.2}$$

The Q and S matrices specify a payoff function in (3.1) which is symmetric in u and may be rewritten as

$$(x_1 + u_1)^2 + (x_2 + u_2)^2 + 2qu_1u_2 \; . \tag{3.3}$$

This payoff function may be interpreted as follows. Each decision maker has a similar task, driving $(x_i + u_i)$ as close to zero as possible. However the interaction term $2qu_1u_2$ gives motivation to coordinate actions. Note that because of the form of the base information, H, if each decision maker knows only his information ($p_{12} = p_{21} = 0$) then each solves the decentralized problem of driving $(x_i + u_i)$ to zero and disregards the interaction term.

The symmetric nature of the problem extends to the characteristics of the decision makers. Each decision maker has the same total precision to spend and the same difficulty reading the other's memos.

The solution to the inner optimization problem is

$$-(1 + p_{11}{}^{-1} - q^2(1 + p_{21}{}^{-1})^{-1})^{-1} \tag{3.4}$$

$$-(1 + p_{22}{}^{-1} - q^2(1 + p_{12}{}^{-1})^{-1})^{-1} \; .$$

To solve this problem, we assume symmetry in the answer: $p_{11} = p_{22}$ and $p_{21} = p_{12}$. Also, we adjoin the constraint on informational precision to the inverse of the part of the payoff in (1.4) that involves p_{11} and p_{21}. Thus the constrained optimization problem is

$$\max_{\lambda} \; [\min_{p_{11},p_{21}} (1 + p_{11}^{-1} - q^2/(1 + p_{21}^{-1})) + \lambda(p_{11} + bp_{21})] \tag{3.5}$$

The solution to this problem is

$$\lambda = (b + c)/(1 + qb^{1/2}) \tag{3.6}$$

$$\left.\begin{array}{ll} p_{11} = p_{22} = \lambda^{-1/2} & \\ & b\lambda \leq q^2 \\ p_{12} = p_{21} = q/(b\lambda)^{1/2} - 1 & \\ & \\ p_{11} = p_{22} = c & \\ & b\lambda \geq q^2 \\ p_{12} = p_{21} = 0 & \end{array}\right. \tag{3.7}$$

Note that the form of the cross information ($p_{12} = p_{21}$) strongly resembles the purchase of information in the one-person case. If we substitute $b\lambda/q^2$ for the price of information, the identical results obtain: no information about the other decision maker's information is demanded if the price is one or greater. Note that, consistent with intuition, the higher the interaction, q, the lower the informational constraint, c, necessary for some interaction to be optimal.

4. Conclusion

Preliminary work on cases more complicated than those considered here indicate similar threshold effects in demand for information obtain in more complex situations. However, the complexity of the solution for the inner optimization problem--the general Linear-Quadratic-Gaussian problem in (1.7)--makes optimization of the information structure and interpretation of the results difficult.

The model does have the distinct advantage of eliminating the problem of combinatorial complexity from problems of calculating optimal information structures. It also has the advantage that the variable precision of the corrupting noise, or equivalently, the variable precision of the information as read, captures an aspect of organizational form--varying degrees of coordination--not captured by discrete choices of information structures.

References

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13. ABSTRACT

Motivated by considerations of optimal organizational form for a firm, a model is constructed that describes an organization as a team each of whose members has access to the same basic information seen through noisy channels. The amounts of noise on the channels are the control parameters of the problem and serve to characterize the organization. The chief advantages of this model are easier calculation of the optimal information structure--organizational form, and an easy way of characterizing bounded rationality by constraining the minimum amount of total noise that a decision maker must accept on his information. For analytic expediency, a general Linear-Quadratic-Gaussian model is constructed and solved for two special cases.

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